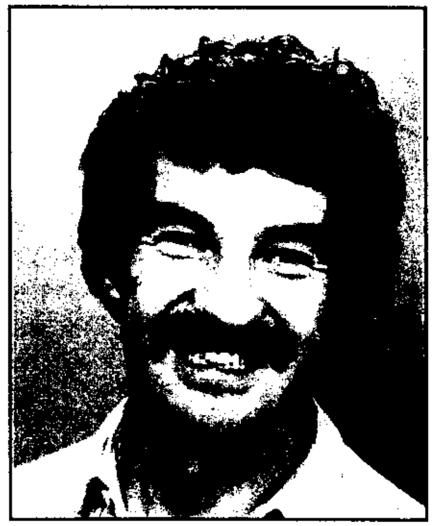
A Simple Estimate of the Mass of the Positron

By Goronwy Tudor Jones

A popular toy consisting of two identical steel pendulums demonstrates that, in an elastic collision, a projectile can transfer all of its momentum to a target of equal mass. The same is true for collisions between relativistic elementary particles.

The exceptional bubble chamber picture of Fig. 1 showing a collision between a positron and a stationary electron enables the mass of the positron to be estimated in a simple way—demonstrating the power and all-pervading nature of the laws of energy and momentum conservation.

Several other esoteric phenomena (some not easy to show on their own) manifest themselves in this picture: the existence of antimatter, pair-creation or the materialization of a high-energy photon into an electron-positron pair; the annihilation of a high-energy positron in flight; the Compton Effect.



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What Is Particle Physics?

The aim of particle physics is to study the fundamental building blocks of nature and the forces they exert on each other. The experimental side of this subject consists of examining what happens when particles are made to collide at high energies at accelerator centers such as CERN (the European Centre for Particle Physics Research) in Geneva or Fermilab just outside Chicago.

In this article, we discuss in isolation a small part of the final state of a high-energy neutrino interaction: a head-on collision of a positron and a stationary electron, something that happens very rarely.

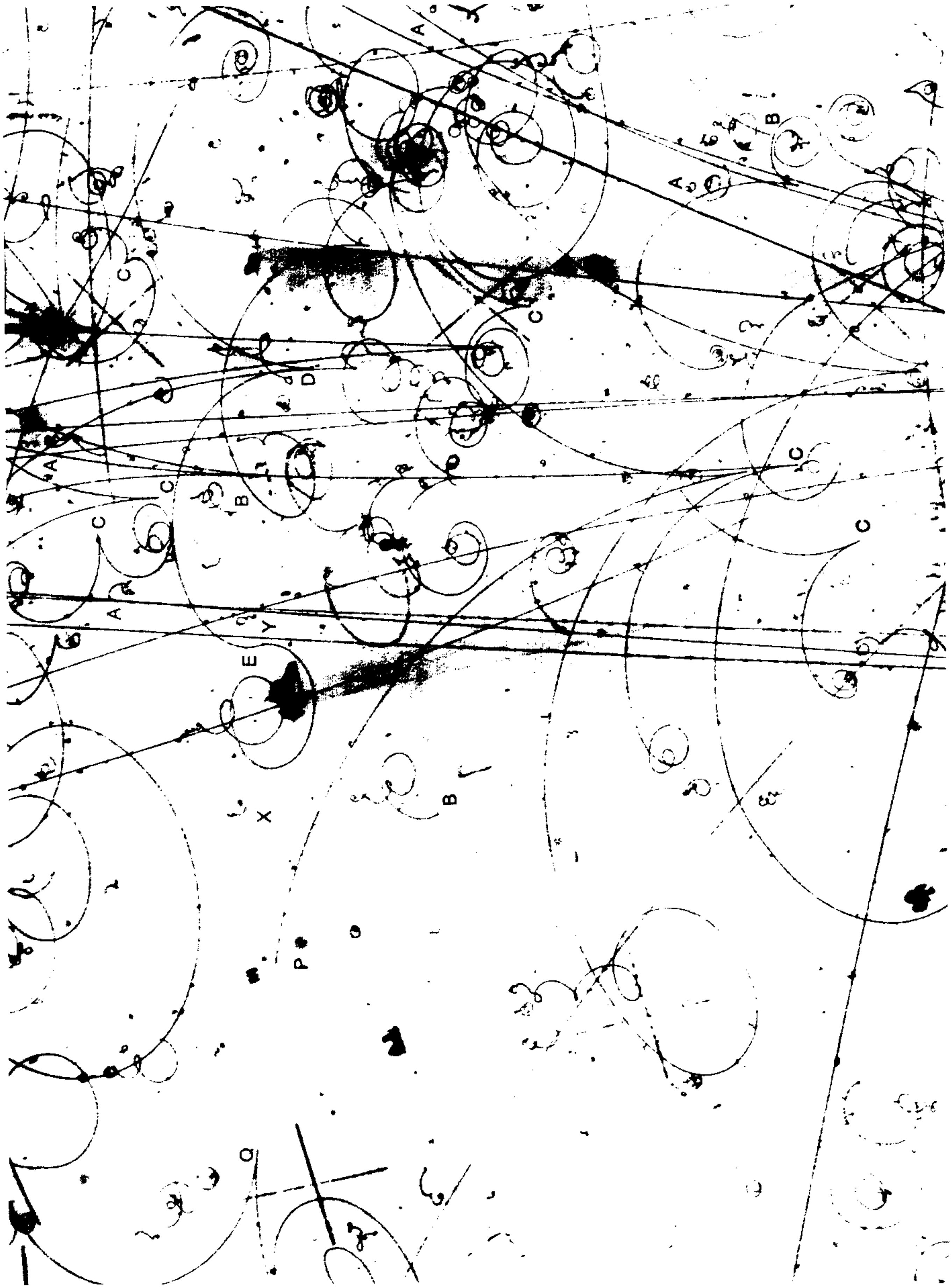
The Bubble Chamber

The bubble chamber[1,2] is a detector that provides data about elementary particle interactions in the form of photographs that are easy to interpret—and beautiful in their own right. The dark lines in Fig. 1 are trails of tiny bubbles that are created as charged particles force their way through the specially prepared liquid of the bubble chamber—in this case a superheated (roughly 2:1) mixture of neon and hydrogen.

As described previously,[1] the tracks are curved by a magnetic field in which the bubble chamber is placed. The curvature of a track can be measured to give the momentum of the particle that produced the track:

$$p = (Bq)r \tag{1}$$

This tells us that for a given field B and charge q, the momentum p is proportional to the radius of curvature r. (For simplicity, we are considering a particle moving at right angles to the magnetic field.) This equation is valid in both nonrelativistic and relativistic situations.



Here, nature has been kind: all charged particles that live long enough to travel a measurable distance have a charge equal or opposite to that on the electron e ($\approx 1.6 \times 10^{-19}$ coulomb). Equation (1) then becomes, in units used by nuclear and particle physicists,

$$p = 0.3Br \qquad (2)$$

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where p is measured in GeV/c, B in tesla, and r in meters. For the Fermilab 15-ft-diameter bubble chamber where this picture was taken, the magnetic field of 3 T would give a 1 GeV/c track a radius of curvature of about 1 m. (At the magnetic north pole, the magnetic field at the Earth's surface is 6.2×10^{-5} T.)

The Bubble-Chamber Picture

Look at Fig. 1 as we describe what we see.

- Several lines coming in from the bottom are diverging; they come from a neutrino interaction, way upstream of the picture. So now we have a feeling for the direction of motion. And the speed? Everything is moving with more or less the speed of light. The ion trails that developed into bubble tracks were created in a few nanoseconds.
- At several places, marked A, a low momentum negative track (it curls to the right) can be seen, beginning on a track of much higher momentum. These are electrons that have been struck by the passing particle that is much more massive than the electron.

  Notice how these electron tracks spiral in, showing that they lose their energy at a considerable rate as they travel. This is in contrast to the massive particles that have struck them. Apart from losing energy by creating bubbles, electrons lose energy much more quickly by another process, known as bremsstrahlung (braking radiation).

  This process, which is a consequence of the fact that all accelerated charges radiate,[3] is important for electrons because they have small masses. One can argue as follows: For a particle of given charge, the amount of energy lost by bremsstrahlung depends on its acceleration; the acceleration is provided by the electric field due to the nuclei of the medium through which the particle is moving; by Newton's second law, the acceleration for a given force varies inversely as the mass. So, since the next lightest charged particle after the electron is the muon, which is over two hundred times more massive, we do not expect much bremsstrahlung from particles other than electrons (especially since it is the square of the mass that counts).

  The upshot of all this is that an electron is instantly recognizable in our bubble chamber because its track will spiral. (This would not be true for a liquid hydrogen bubble chamber because the singly charged hydrogen nuclei do not produce enough acceleration.)
- At several points, marked B, a lone (spiraling) electron can be seen. This is an electron that has been knocked out of an atom by a high-energy photon, or γ-ray. The photon does not leave a track because it is electrically neutral. Such electrons are called Compton electrons. There are several more in the picture. Can you find them?
- There are several points, marked C (two special ones we shall return to later are marked D and Q), from which two spiraling tracks, one positive and one negative, are seen to emerge with zero opening angle. These are high-energy photons materializing, in the field of a nucleus, into a positron-electron ($e^+ e^-$) pair. The positron is the antiparticle of the electron.

  The first thing to note is that the positron tracks look pretty much like the electron tracks apart from curving in the opposite direction: they leave trails of bubbles, they spiral; there is nothing mystical about antimatter.
- At P, a positron in flight annihilates with an electron. The photon that is produced materializes at Q, which is ~ 10 cm away from P, along the line of flight.

  This is a classic signal of a positron, but even bubble-chamber physicists do not see them often; the positrons usually slow down and stop before annihilating.
- Now we come to another special feature of this picture, and the main point of interest of this article. At the point marked E, the positron track that left D seems to change into a negative track of more or less the same curvature (momentum). What has happened is that the positron has made a head-on collision with an electron, transferring what looks like all its momentum to the electron—suggesting that the mass of the positron is equal to that of the electron. Such an occurrence is very rare in bubble-chamber physics.

We now discuss in detail the kinematics of this problem, present the measurements of the curvatures of the tracks on either side of the collision point E, and see what limit we can set on the mass of the positron.

Fig. 1. (See facing page.) Part of a bubble-chamber picture. Dark lines are trails of tiny bubbles created as charged particles force their way through a tank of transparent liquid enclosed in a powerful magnet. At E, the main point of interest, a positive track curling to the left seems to change its sign. What is happening is that a positron collides head-on with a stationary electron and loses all (within errors) its momentum. This shows that (within errors) the positron has the same mass as the electron. (This picture, from neutrino experiment E632 performed at the Fermilab 15-ft bubble chamber, was found at the University of Birmingham.)

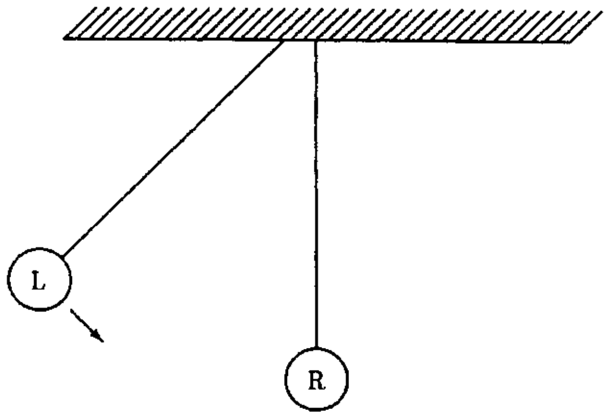


Fig. 2. Part of a popular toy known as *Newton's Cradle*.

Nonrelativistic Elastic Head-on Collisions

Figure 2 shows part of a coffee-table toy known as Newton's Cradle. If the left (L) pendulum of mass m_L is pulled to one side and let go so that it collides with the stationary right (R) one of mass $m_R = m_L$, the latter will rise to the height from which the former was released, provided the collision is elastic. We shall see that this follows from momentum and energy conservation laws, which say that in such a collision all the momentum is transferred from the projectile to the target.

If the initial and final speeds are denoted by u and v respectively, momentum conservation gives

$$m_L u_L + m_R \times 0 = m_L v_L + m_R v_R \tag{3}$$

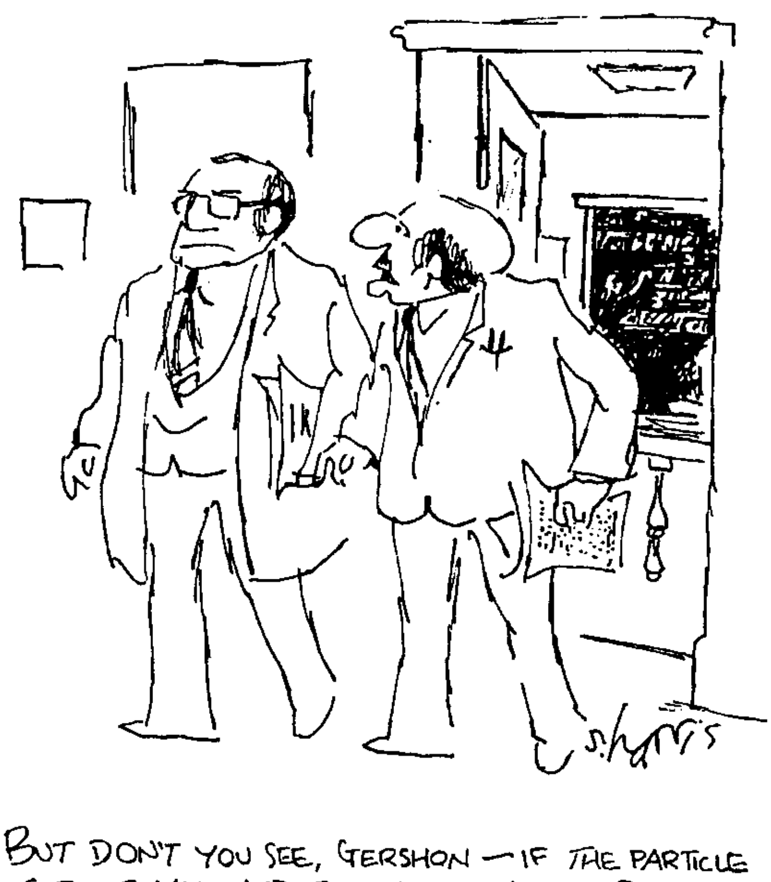
and since, by definition, kinetic energy is conserved in an elastic collision,

$$\frac{1}{2}m_L u_L^2 + \frac{1}{2}m_R \times 0^2 = \frac{1}{2}m_L v_L^2 + \frac{1}{2}m_R v_R^2 \tag{4}$$

For the equal mass situation we are considering at the moment, these equations reduce to $u_L = v_L + v_R$ and $u_L^2 = v_L^2 + v_R^2$. Equating values of v_L^2 from each yields $v_R^2 = u_L v_R$, which has two solutions:

- $v_R = 0$; this corresponds to there being no collision; ball L was not shot directly at ball R.
- $v_R = u_L$; this, as anticipated, corresponds to a complete transfer of L's momentum to R.

Let us now investigate what fraction of its momentum a mass m_L (> m_R) can impart to one of mass m_R. That it is less



"BUT DON'T YOU SEE, GERSHON — IF THE PARTICLE IS TOO SMALL AND TOO SHORT-LIVED TO DETECT, WE CAN'T JUST TAKE IT ON FAITH THAT YOU'VE DISCOVERED IT."

than the whole can be appreciated by thinking about the case of L being much more massive than R.

If we define $\lambda = m_R/m_L$ then Eqs. (3) and (4) can be rewritten:

$$v_L = u_L - \lambda v_R \tag{5}$$

and

$$v_L^2 = u_L^2 - \lambda v_R^2 \tag{6}$$

Equating values of v_L^2 we get

$$v_R = \frac{2}{\lambda + 1} u_L \tag{7}$$

Let us now use this formula to ask: If the mass of the projectile L is n times that of target R, what fraction of its momentum can be transferred to R?

From Eq. (7) we see that the speed with which R moves is given by

$$v_R = \frac{2}{\frac{1}{n} + 1} u_L \tag{8}$$

The momentum of R is then given by multiplying by its mass, $\frac{m_L}{n}$:

$$\text{Momentum of struck ball} =$$

$$\frac{m_L}{n} \times \frac{2n}{n+1} \times u_L = \frac{2}{n+1} \times m_L u_L \tag{9}$$

So, projectiles of 2, 3, and 5 times the mass of a target will, in a head-on elastic collision, transfer only $2/3$, $1/2$, and $1/3$ of their momenta respectively. Note that this result applies, no matter what the energy of the projectile. The same is not true for the relativistic case. Before moving on to this, let us remind ourselves of the problem in hand.

Details of the Experimental Measurement

The bubble-chamber picture of the positron transferring all its momentum to the stationary electron suggests, if nonrelativistic mechanics is any guide, that the positron mass equals that of the electron. The first thing we need to show is that a proper relativistic treatment of the collision would lead to the same inference. It will be shown to be so.

Then we have to worry about reality—we cannot measure momenta without errors. In bubble-chamber picture analysis, the momentum of a track is obtained by measuring, on at least two views so as to be able to reconstruct in three dimensions, the coordinates of several points along a track. Because of the measurement errors, these points will not lie on a perfectly smooth curve. The curve that best fits through these points is then calculated, together with errors that give a feel for the spread of curves that could be considered consistent with the measured points. The radius of curvature (with error) of this curve gives via Eq. (2), the momentum of the track (with error).

In principle this is straightforward. In practice it is a complicated procedure. For one thing, the particles lose energy as they force their way through the bubble-chamber liquid; so they are more curved at their ends than they are at their beginnings; this must be taken into account.

In the case of light electrons, curvature changes due to bremsstrahlung are unpredictable and often quite severe, making a momentum measurement particularly difficult.

Using the current state-of-the-art fitting program from CERN, our measurements for the e^+ approaching point E on the picture, and the e^- leaving it, give 54 ± 15 and 53 ± 13 MeV/c respectively. (Systematic errors are negligible in comparison with these large statistical errors, forced upon us by the short lengths of track that can sensibly be used for measurement, typically 3 cm.)

The fraction of the positron's momentum transferred to the electron is then 0.98 ± 0.37, where the errors have been added in quadrature.

Relativistic Elastic Head-on Collisions

Consider a projectile of mass M, energy E_M, and momentum p_M making a head-on elastic collision with an electron of mass m_e ($< M$). Let the corresponding final state energies and momenta be E_M^f, p_M^f, and E_e, p_e as shown in Fig. 3. (Since we are considering a head-on collision, momentum is a one-component vector.)

Fig. 3. A projectile of mass M, energy E_M and momentum p_M makes a head-on collision with an electron of mass m_e. In the final state, the energies and momenta are given by E_M^f, p_M^f, and E_e, p_e.

Momentum and energy conservation give

$$p_M = p_M^f + p_e \tag{10}$$

$$E_M + m_e c^2 = \sqrt{p_M^{f\,2} c^2 + M^2 c^4} + \sqrt{p_e^2 c^2 + m_e^2 c^4} \tag{11}$$

Eliminating p_M^f we get

$$E_M + m_e c^2 - \sqrt{p_e^2 c^2 + m_e^2 c^4} = \sqrt{(p_M - p_e)^2 c^2 + M^2 c^4} \tag{12}$$

Squaring and then subtracting $p_e^2 c^2$ from both sides gives

$$(E_M + m_e c^2)^2 + m_e^2 c^4 - 2(E_M + m_e c^2)\sqrt{p_e^2 c^2 + m_e^2 c^4} = p_M^2 c^2 - 2p_M p_e c^2 + M^2 c^4 \tag{13}$$

We now have a fair bit of algebra ahead of us. This can be simplified by noticing how c appears in Eq. (13): there is always a c multiplying a momentum, and a c^2 multiplying a mass. Let us therefore forget these c's for now, and bring them back at the end. (This is what particle physicists call the $c = 1$ convention.)

Notice: under this convention, the relativistic expression that relates the rest mass m_0 of a particle to its energy and momentum, $E^2 = p^2 c^2 + m_0 2 c^4$, reduces to $E^2 = p^2 + m_0 2$; it looks strange at the moment, but don't worry, we know where the c's should be and will put them back.

We now have

$$(E_M + m_e)^2 + m_e^2 - 2(E_M + m_e)\sqrt{p_e^2 + m_e^2} = p_M^2 - 2p_M p_e + M^2 \tag{14}$$

Evaluating the square, using $E_M^2 - p_M^2 = M^2$, rearranging and dividing by 2, we get

$$m_e^2 + E_M m_e + p_M p_e = (E_M + m_e)\sqrt{p_e^2 + m_e^2} \tag{15}$$

Squaring, removing terms appearing on both sides of the equation, and rearranging, we get

$$p_e(p_M^2 p_e + 2p_M m_e^2 + 2E_M m_e p_M - E_M^2 p_e - m_e^2 p_e - 2E_M m_e p_e) = 0 \tag{16}$$

(The solution $p_e = 0$ corresponds to there being no collision—the momentum conservation equation does not explicitly say that the projectile was fired straight at the electron.) Solving for p_e we get, after making use of $E_M^2 - p_M^2 = M^2$,

$$p_e = \frac{2p_M m_e (E_M + m_e)}{M^2 + m_e^2 + 2E_M m_e} \tag{17}$$

Putting the c's back in we now have an expression for the fraction of the projectile's momentum that is imparted to the electron:

$$\frac{p_e}{p_M} = \frac{E_M + m_e c^2}{E_M + \frac{m_e c^2}{2}\left(1 + \frac{M^2 c^4}{m_e^2 c^4}\right)} \tag{18}$$

Writing $M = nm$ we get the relativistic version of Eq. (9):

$$\frac{p_e}{p_M} = \frac{E_M + m_e c^2}{E_M + \frac{n^2 + 1}{2} m_e c^2} \tag{19}$$

We notice immediately that, for the equal-mass case ($n = 1$), the target picks up all the projectile's momentum, just as we found for the non-relativistic case $\left(\frac{p_e}{p_M} = \frac{2}{n+1}\right)$.

However, for $M > m_e$, the fraction imparted depends on the energy E_M of the projectile. Figure 4 shows a plot of this fraction f as a function of n for a projectile of energy $E_M = 54$ MeV (so long as the positron is highly relativistic and its energy is then given by $E_M = pc$, with $p = 54$ MeV/c).

The horizontal dotted line is drawn at $f = 0.24$, two standard deviations from the measured value of 0.98 ± 0.37. (See last sentence on p. 99.) This meets the curve at $n = 26$. In the jargon of the statistician, we can then say that, at the 95% confidence level, the mass of the positron is less than 26 electron masses.

This method of seeing that the positron has a low mass (compared with the proton, say) has the virtue of clarity and beauty, but it falls a long way short of the current best estimate of the mass difference between the e^+ and the e^-. Steven Chu et al.[4] measured with very high accuracy the frequency of radiation emitted from positronium (an e^+e^- bound state), and quote

$$|m_{e^+} - m_{e^-}| < 4 \times 10^{-8} \quad (20)$$

at a 90% confidence level.

Finally: What happened to the positron that was stopped by the electron?

If, indeed, the positron had been stopped dead in the collision, it would then have been annihilated with an electron to produce two 0.511-MeV photons, back to back, just as in Positron Emission Tomography (PET) scanners in hospitals. Such low-energy photons cannot materialize into electron-positron pairs; this would violate energy conservation. However, our measurement errors allow the positron to *keep* ~ 10 MeV in the bubble chamber, which could be passed on to the annihilation photons. If we look at Fig. 1 very carefully, we see two pieces of evidence that could result from these annihilation photons, marked X and Y on the picture.

- X is a Compton electron with a momentum of 7 ± 2 MeV/c that maintains its position relative to the collision point E on all camera views. It may reasonably be interpreted as having been produced by one of our annihilation photons.
- Y looks like an even better candidate, a materializing e^+e^- pair *pointing back* towards E. Unfortunately, it does not maintain its orientation relative to E and is therefore spurious. Sad!

The information provided here, in addition to being of interest in its own right as a simple measurement of the mass of the positron, can be of value to teachers of more traditional courses by providing illustrations of basic concepts such as conservation laws and the emission of electromagnetic radiation by accelerating charges.

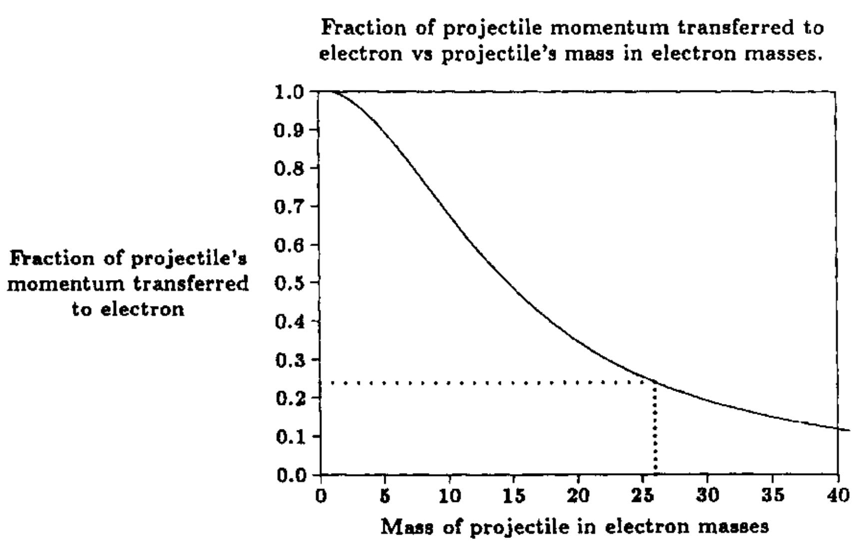


Fig. 4. Plot showing, for a relativistic head-on elastic collision between a projectile of mass $M = nm_e$ and a stationary electron of mass m_e, how the fraction of the projectile's momentum that is lost to the electron depends on n.

Acknowledgments

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2. Clifford E. Swartz, "Believing is seeing," *Phys. Teach.* **30**, 9 (1992).
3. All the electromagnetic radiation in which we are bathed—sunlight, radio waves, etc.—started off somewhere, sometime, as a result of charges accelerating.
4. Steven Chu et al., "Measurement of the positronium $1^3S_1 - 2^3S_1$ interval by Doppler-free two-photon spectroscopy," *Phys. Rev. Lett.* **52**, 1689 (1984).