Why is the bubble-density higher for slow tracks?

* INTRODUCTION

When a massive (m>>m_e) charged particle travels through matter (a bubble chamber liquid, for example), it exerts Coulomb forces on the electrons of the atoms, imparting momentum to these electrons.

In high energy physics we deal with particles of energies from a few hundred MeV to hundreds of GeV – enormous compared to the few eV needed to ionize atoms.

From $E^2 = p^2c^2 + m^2c^4$ we see that when the momentum (in GeV/c) become more than a few times the rest mass (in GeV/c^2), the particle is relativistic: $E \sim pc$.
In our bubble chamber pictures, such relativistic particles are `minimum ionizing'.

Exercise: Many of our bubble chamber pictures involve a K^- beam with an energy of 4.2 GeV. What is the velocity of a 4.2 GeV kaon? Would it be minimum ionizing?
Answer:
By the following calculation:

$$m_K c^2 \approx 0.5\,\mathrm{GeV} \Rightarrow {m_K}^2 c^4 \approx 0.25\,\mathrm{GeV}^2$$
$$E_K^2 \approx (4.2\ \ \mathrm{GeV})^2 = 17.6\,\mathrm{GeV}^2$$
$$\therefore\ p_K^2 c^2 = E_K^2 - {m_K}^2 c^4 = 17.35\,\mathrm{GeV}^2 >> {m_K}^2 c^4$$

We see that the K$^-$ is highly relativistic(v~c) and therefore minimum ionizing.

* NUMBER OF BUBBLES/CENTIMETRE ~ $1/v^2$.

Now let us return to our massive (m>>m_e) charged particle travelling through matter, and try to get a feel for why the rate at which it loses energy (number of bubbles per centimetre, or, to use the jargon of the particle physicist, `ionization density', dE/dx) is inversely proportional to v^2.

At any point along its trajectory, the force exerted on the electron by the projectile can be resolved into components parallel and perpendicular to the direction of motion of the projectile. For every point on the trajectory to the left of the point of closest approach there is a corresponding point to the right, for which the perpendicular component of force on the electron is the same, but the parallel component is in the opposite direction. So to work out what happens to the electron, we need only consider the component of force at right angles to the projectile's motion.

A simple first approach is to use Newton's second law: the time-integral of a force (its impulse I) equals the change in momentum. If the force acts for a short time we can say

$$I \sim \text{average force} \times \text{its duration}$$

To estimate the impulse imparted to the electron by the massive projectile of charge Z and distance of closest approach b, we picture it as feeling a force $Ze \cdot e / 4\pi\varepsilon_0 b^2$ (an overestimate because this is the maximum force) for a time $2b/v$ (an underestimate, because this is the time for which the force is between $0.5F_{max}$ and F_{max}).

We then have an expression for the momentum p imparted to the electron:

$$p = (Ze \cdot e / 4\pi\varepsilon_0 b^2) \times 2b / v \sim 1/v$$

Hence, the kinetic energy picked up ($p^2/2m$) ~ $1/v^2$, as required.

(For a detailed derivation, click here. This topic was first discussed in visionary papers by Niels Bohr: *Phil. Mag*. 25, 10 (1913) and *Phil. Mag*. 30, 581 (1915).)