Astronomical Telescopes

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1 Telescope Overview

An astronomical telescope is an optical tool to capture light and deliver it to instruments that measure the amount of light coming from different directions, its spectral content, and its polarization. There are usually several limiting factors determing the performance of the telescope at these tasks

- Earth's atmosphere
- Telescope aperture
- Optical quality
- Diffraction
- Detector design
- Spectrograph and polarimeter design
- Photon statistics

to mention the most signficant. Let's take these one by one.

Earth's atmosphere

At any altitude a human astronomer could work, the air we breath absorbs light that would otherwise be interesting for astronomy. This absorption includes all wavelengths below about 350 nm (3500 Å), wavelengths in the infrared above 1000 nm (1 μm or 10,000 Å) excepting specfic bands between molecular absorption in the infrared, and longer wavelengths that are in the radio frequency regime. Ground based optical telescopes are therefore designed for optimal work in the visible and near infrared in these standard filter bands

Johnson-Cousins U B V R I from 365 nm to 787 nm

Sloan u g r i z from 354 nm to 905 nm

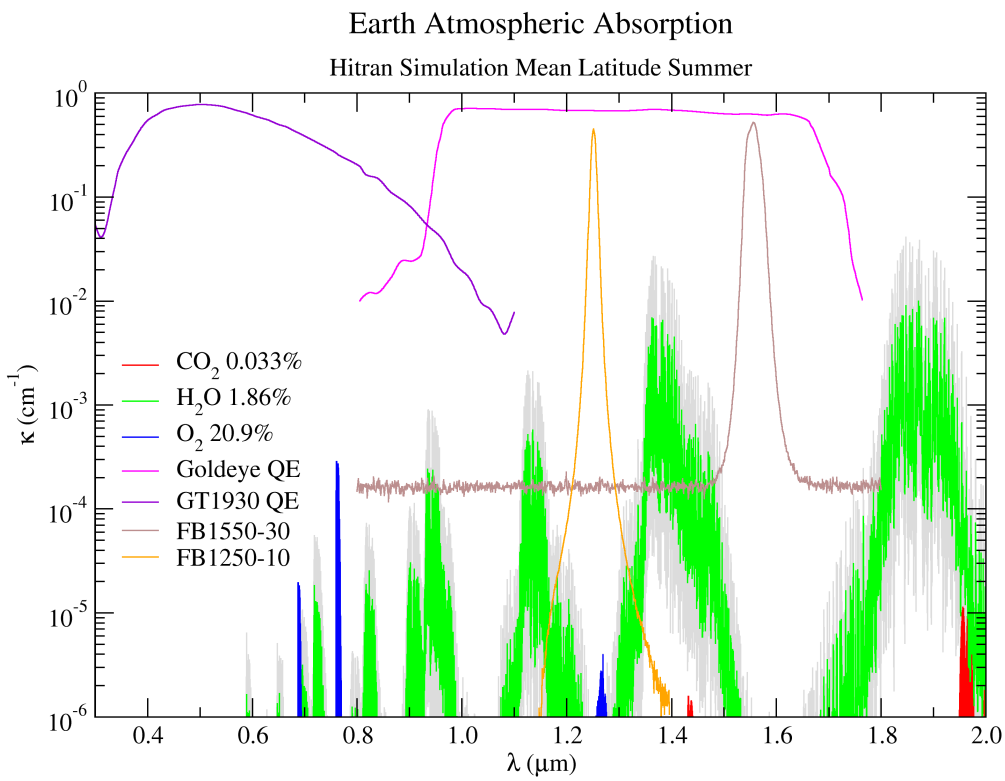

Figure 1: Transmission of Earth's atmosphere with the response of silicon and InGaAs sensors.

Infrared I J H K L from 787 nm to 3450 nm

The atmosphere also scatters light, introducing stray light from urban lighting and the Moon, emits light of its own that we term airglow, and distorts imaging. Flowing air with density variations is often turbulent, and the variations in gas density along the optical path, both at altitude and within the optical system, cause wavefront deviations that usually exceed the errors in the optics and diffraction. Turbulence, what astronomers call "seeing", limits the image quality severely.

Telescope aperture

Today's "small" telescope is last century's giant, and even instruments in the class of 4 meters diameter are dwarfed by those under development. This is not to say that apertures measured in centimeters rather than meters are not useful, but that the science that can be done with them is in a different realm. The very largest telescopes gather light enabling

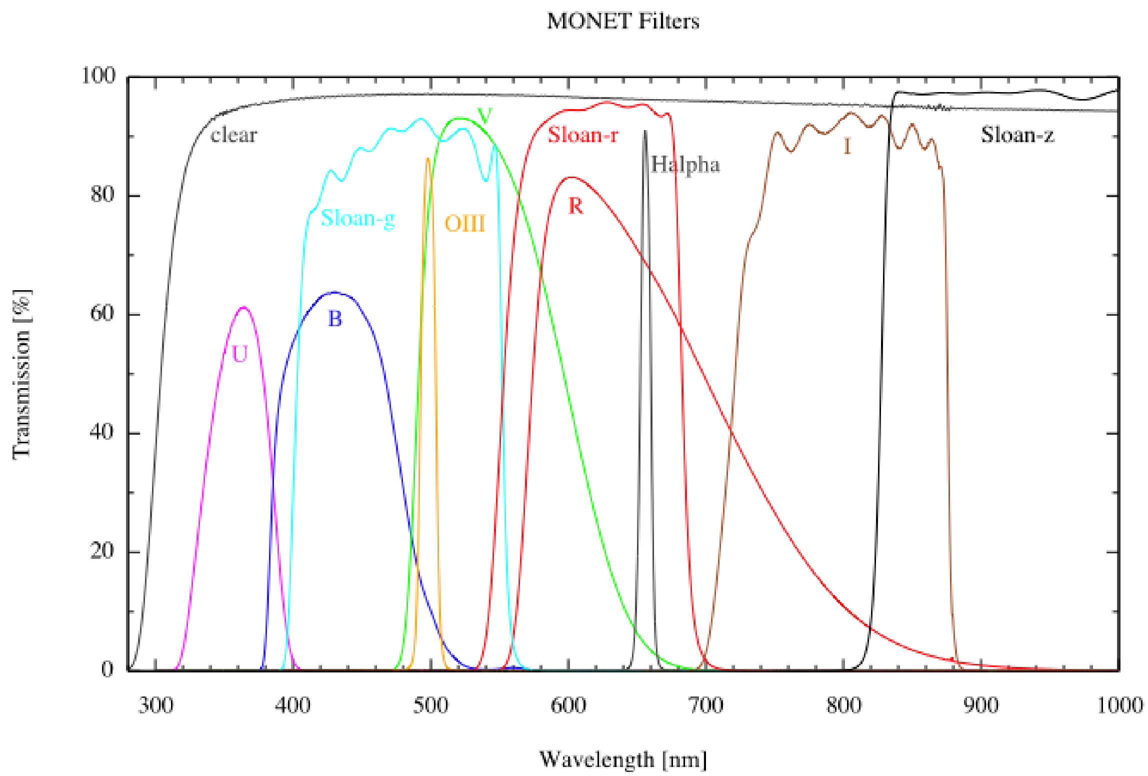

Figure 2: Standard astronomical filters for the visible and near-infrared as used by the Monet telescope. Credit: Frederick Hessmann.

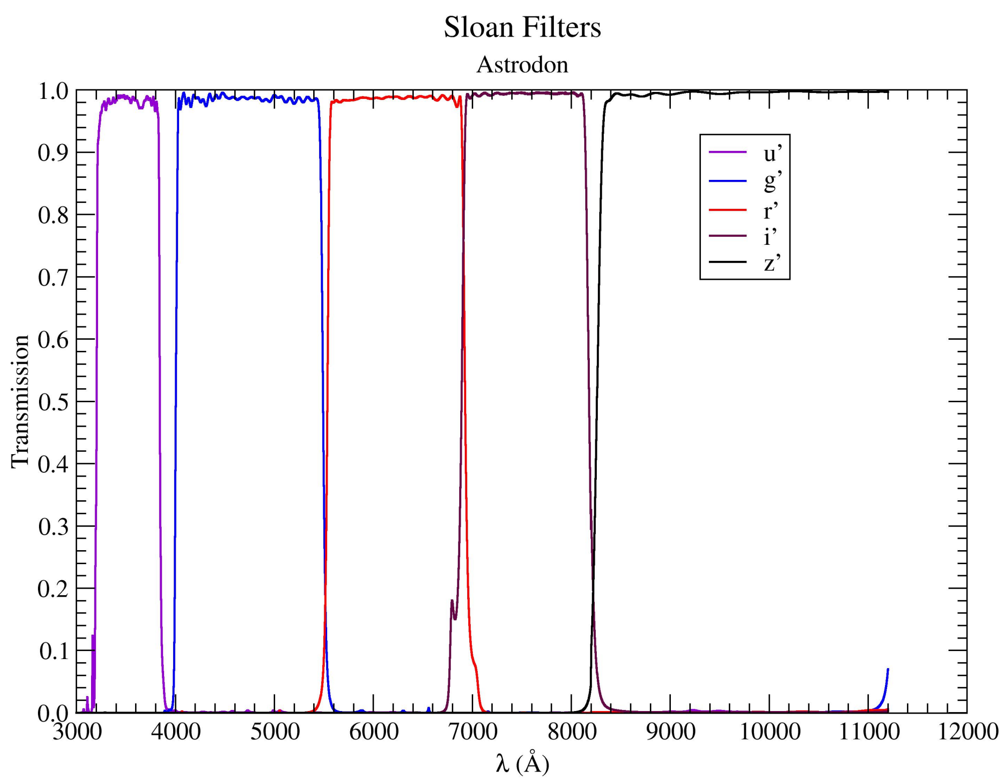

Figure 3: Sloan filter set.

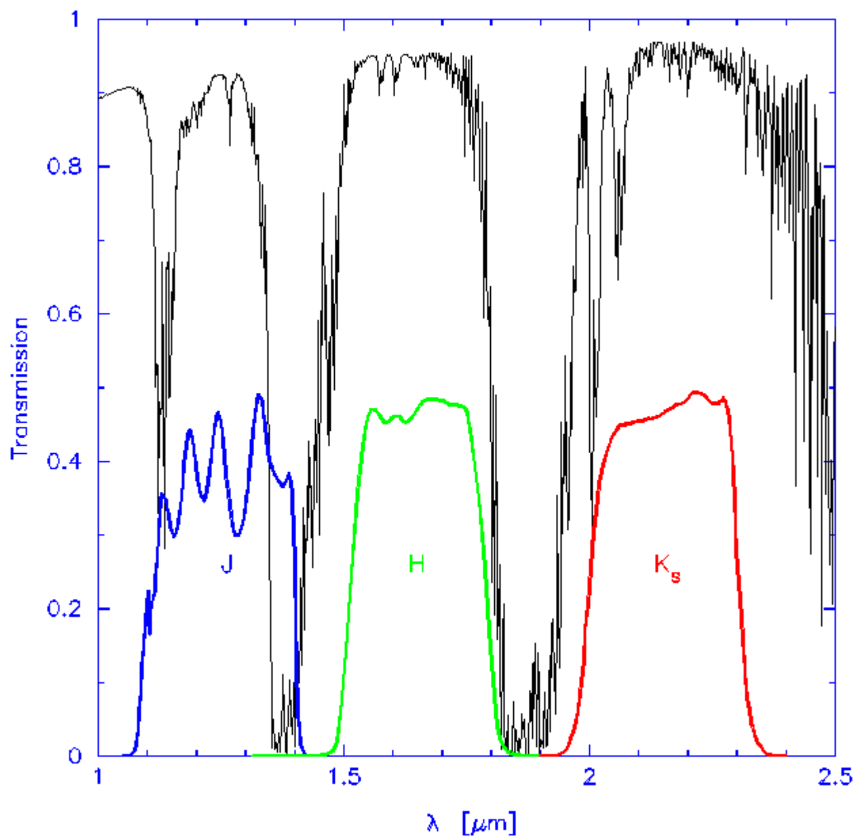

Figure 4: JHK near-infrared filter set.

studies of the faintest and most distant objects of interest. Smaller telescopes offer wide fields of view, and are appropriate for brighter closer objects. They are also naturally less subject to turbulence, and capable of high resolution imaging without adaptive controls to correct for the atmosphere, and their affordability means they can dedicate time to long duration studies. Thus telescopes under 35 cm might be used by amateur astronomers for their work, and for education. Telescopes in the 0.5 to 2.5 m class are often dedicated to specific areas of research that require rapidly cadenced data on objects brigher than 18th magnitude, or on spectroscopy of bright stars. Larger telescopes would primarily be directed to fainter and transient events – distant galaxies, supernovae, faint star spectroscopy, and small solar system objects. Currently the pair of Keck telescopes in Hawaii each with 10 m mosaic mirrors lead the aperture race. However ESO has 4 very large telescopes (VLT) with 8.4 m diameter mirrors in Chile, the Gemini telescopes at 8.1 m cover the northern and southern sky from Mauna Kea and Chile, and many others have apertures in the 3 to 6 m range. The European Extremely Large Telescope (ELT) will be operating with a 39 m aperture by 2024, and before it the Vera Rubin formerly known as the Large Synoptic Survey Telescope or LSST, at 8.4 m, will have first light by 2022. Others in this class are planned, or under construction. Because of the very high cost, most of these are operated by national governments or private consortia, and all have very limited access based on membership and competitive proposals. By contrast, Vera Rubin when it is running will produce terabytes of data per night that will be available to astronomers in the U.S.

Telescopes in space range from the James Webb Space Telescope (JWST) to be launched in 2021 with a 6.5 m aperture, the Hubble Space Telescope (HST) at 2.4 m, Gaia at 1.45 m, the now-decomissioned Kepler spacecraft at 0.95 m, and the Transiting Exoplanet Survey Satellite (TESS) launched in 2018 at a tiny 10 cm for each of its four cameras.

Typically larger apertures must be made with reflecting optics, supplemented in the optical path by refracting elements to correct the images. The technical reason for this is that reflecting optics have one surface that can be supported from the back, and that segmenting the optics into multiple components to achieve very large diameters is feasible. Reflecting optics are also achromatic and will focus equally light from the ultraviolet to the infrared. Refracting optics in smaller apertures are valuable for their favorable focal length to aperture ratio (the f-ratio), and for their ability to image very wide fields with good quality. Almost all large optical telescopes incorporate refracting elements to correct for aberrations, though some designs work well with mirrors alone for small fields of view.

Optical quality

Optical elements work by modifying the plane wavefront of light from a distant object so that it arrives coherently at a focus where a sensor, each pixel of the camera, can detect it and provide a measurable signal for analysis. If the optical element is not perfect, the wavefront that leaves it will not converge to a single point, and the image of a distant point source will spread over pixels and be blurry. The conventional standard is that the surfaces must preserve the wavefront to within $\lambda/4$, so-called "quarter-wave" optics, to avoid this

Figure 5: University of Louisville's "Shared Skies" CDK20-North telescope at Moore Observatory.

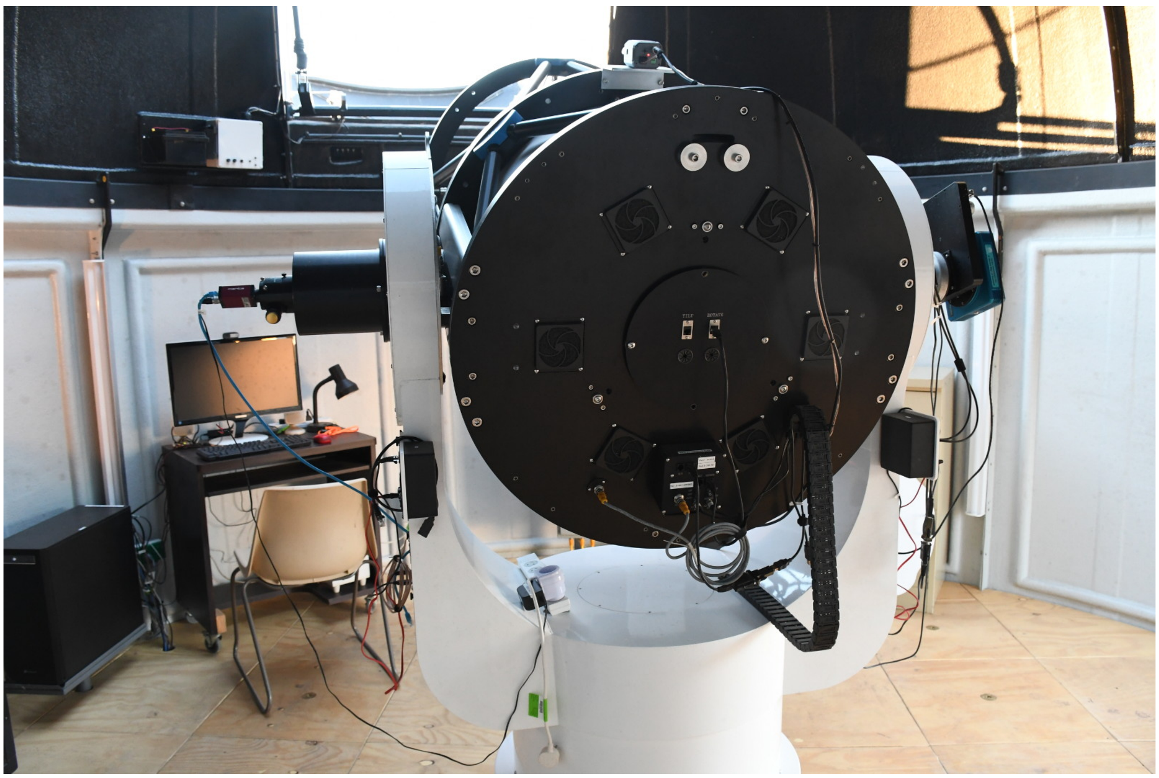

Figure 6: University of Louisville's "Shared Skies" CDK700 telescope at the University of Southern Queensland's Mt. Kent Observatory in Australia.

Figure 7: University of Louisville's "Manner Telescope", an 0.6-meter Ritchie-Chrétein located at the University of Arizona's Stewart Observatory on Mt. Lemmon near Tucson.

outcome. However for the best performance and minimizing the spread of the light outside the diffraction pattern, $\lambda/20$ is a more desirable constraint. Such precision is a challenge to achieve in any optical element, and especially a very large one. For the extremely large telescopes in use today, some adaptive control of the optical surface is required to meet this high standard. Since monolithic mirrors in apertures of several meters are being made that are of this quality, they may be combined in a mosaic to make larger instruments when supported by actuators that respond to measurements of wavefront error.

Diffraction

Ultimately the ability of a telescope to resolve detail is limited by diffraction. For at telescope of diameter d in meters, the light spreads over an angle of approximately $0.1/d$ arcseconds. Thus a telescope with an aperture of 10 cm, that is 0.1 meter, resolves about 1 second of arc. In principle, a 10 m diameter telescope will resolve 0.01 arcseconds or 10 milli-arcseconds. This is not achieved in practice from the ground because of optical defects and atmospheric turbulence, but both can be mitigated by actively adapting the optics to correct for image errors.

Detector design

For astronomical imaging, the telescope focuses light onto a detector that is typically a charge coupled device with elements that are several microns (1 μm is 10^{-6} m) across. Each imaging sensor may contain 4096×4096 pixels or more, and cover an area of 50 mm on a side. At the detector, the focal plane scale is set by simple geometry. If x is a distance in the focal plane and f is the focal length, the angle imaged on x is

$$\theta = \arcsin(x/f) \approx x/f \tag{1}$$

From this it follows that for a fixed distance set by the detectors structure, the angle on the sky is smaller the larger the focal length. The area of the sky covered by the detector is also smaller the larger the focal length. Telescope designs constrain the ratio of focal length to aperture to values usually greater than 2:1 (an f/2 optical system), and more typically in the largest telescopes to 8:1 or more. Auxilliary refracting optics may speed up the system. However, the largest telescopes still require very large detector arrays to cover a reasonable field of view. For more modest optics, the telescope's imaging field is covered by a single detector. As an example, a 0.5 meter telescope with a focal ratio of 7 would have a focal length of 3.5 meters. The focal plane scale is 59 arcsecond/mm, and a 10 μm pixel will resolve 0.59 arcsecond. That is, the atmospheric blurring will spread light over an area about 2 pixels wide, while 4096 pixels will span 40 arcminutes. That's larger than the full Moon.

Spectrograph and polarimeter design

Telescopes may also be used to feed light into other instruments, most commonly spectrographs to disperse the light into components, and polarimeters to enable precise determination of polarization. These instruments have their own constraints. For example, polarimeters are sensitive to any element of the optics that may change the polarization state of the light, and therefore reflections that are not normal to the surface of the optic are undesirable. Spectrographs require very small entrance apertures to limit the spectral range, and very long focal ratios to minimize aberrations. When coupled with fiber optics, the size of the fiber must match on one end the characteristics of the spectrograph, and on the other the characteristics of the telescope. As a consequence, the cost and complexity of a spectrograph may rival that of the telescope itself, and may determine the preferred telescope design.

Photon statistics

Lastly, the measurement of light from faint sources is determined by the precision with which the number of photons can be determined. Photon flux is a Gaussian random process, and the standard deviation in counting N photons is $\pm\sqrt{N}$. Thus the signal-to-noise ratio is also $\sqrt{N}$ and to determine a signal to 1% requires 10^4 photons detected. Larger telescopes collect proportionally more photons, and as a practical matter with an aperture of 0.5 m stars as faint as 20th magnitude can be measured. To go fainter it takes more light and therefore more telescope area. A telescope 10 times larger in diameter has 100 times more area and collects 100 times more light. A factor of 100 in light is 5 magnitudes on the astronomical scale. Weighing in with this simple math, the airglow also contributes at the level of a 20th magnitude star per arcsecond, so sites with less airglow and better seeing, allowing smaller image blur, are favored for detecting the faintest, most distant objects.

2 Telescope Optics

Typically an astronomical telescope uses large reflecting optical surfaces to collect light and bring it to a focus on a detector. For visible light imaging in affordable smaller telescopes, this is achieved by using a primary concave mirror and a secondary convex mirror paired with non-spherical surfaces to produce the best image quality at the focal plane for the intended purpose. A commercial design known as a "corrected" Dall-Kirkham Cassegrain system is shown in Figure 8.

In the Dall-Kirkham design the primary mirror is ellipsoidal and the secondary mirror is spherical. The curvatures are chosen to minimize coma and spherical aberrations. The transmission corrector optics shown in the figure improve the overall image quality out to a wide field of view, and enable fast (small ratio of focal length to aperture) optics that reduce the length of the telescope. They also flatten the field of view to match today's detectors. These so-called CDK instruments are now widely used in telescopes up to 2 meter in aperture. [1] An analysis of the geometrical optics of this design is shown in Figure 9

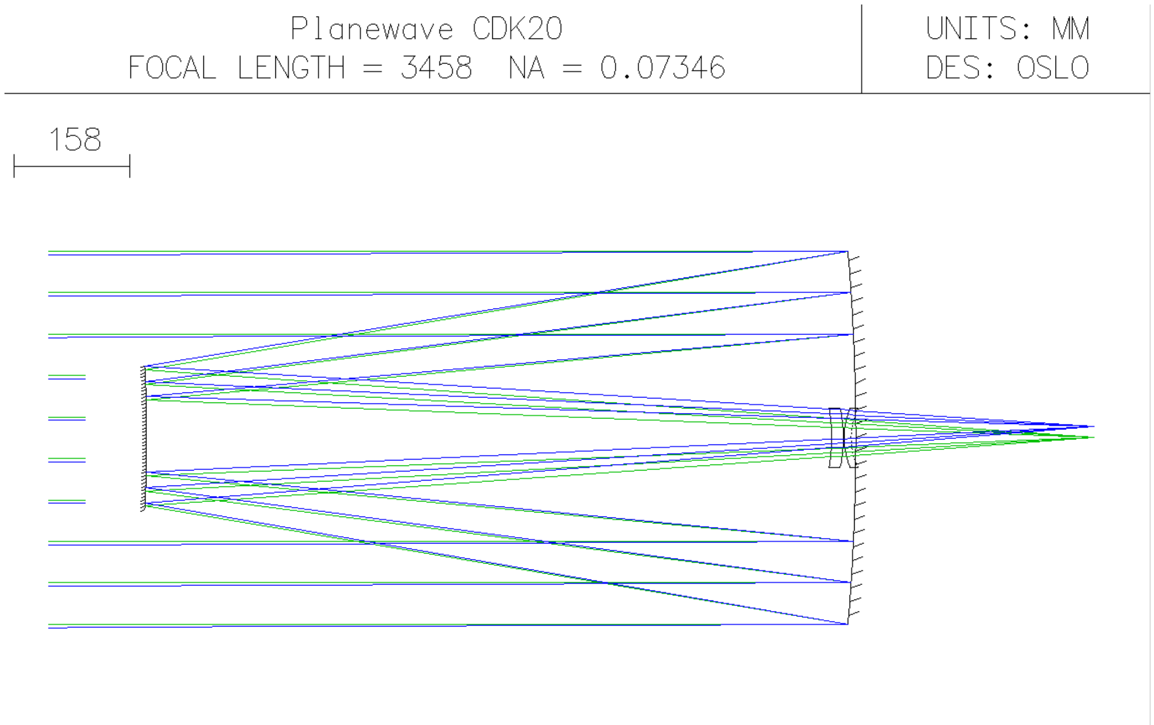

Figure 8: Ray tracing a cross-section of a 0.5 meter corrected Dall-Kirkham telescope.

For larger research instruments it is common to use an aspheric primary and secondary in a Ritchie-Chrétien design, which also produces high quality images but without the transmission optics. Such RC telescopes have poorer performance off-axis than the CDK, and because of the aspheric surfaces are more expensive to manufacture. However without the transmission optics they can reach into the near ultraviolet, or into the infrared.

3 Diffraction

Light leaves a distant source with the properties of a spherical wave. That is, the phase of the wave is constant on the surface of any sphere surrounding the source. When the radius of the sphere is large, in a small enough local region it would be indistinguishable from a plane. Thus, a wave arriving from a star at a telescope on Earth is described as a plane wave. We consider the special case of the wave incident exactly on the optical axis, which is to say that the telescope is pointed precisely at the star.

The entrance aperture of the telescope has an area A that usually would be circular. Because the area limits the light that can go forward into the optics, the wave that propagates through the optical system is modified by diffraction. In the conventional description of diffraction of a wave we neglect the effects of polarization and quantum optics, and simply sum all of the sources of amplitude over the aperture. This description is known as Kirchhoff's

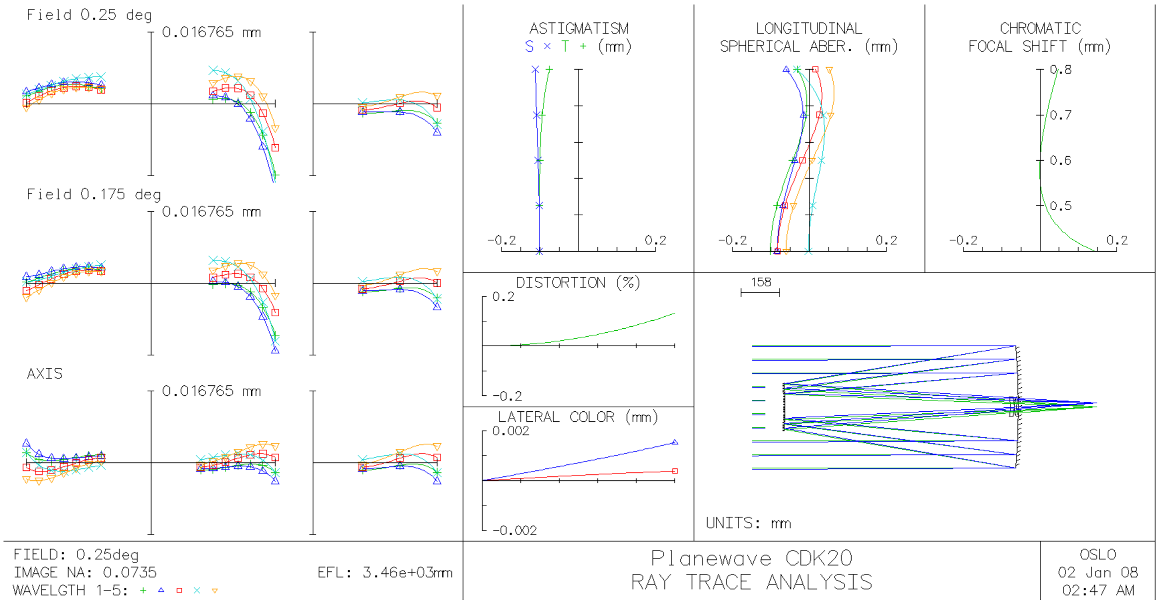

Figure 9: Analyis of the image quality a 0.5 meter corrected Dall-Kirkham telescope.

diffraction theory, in which the amplitude of the wave into an arbitrary direction after the aperture is given by [2]

$$U(P) = C \int \int_A \exp(-iks) dS \qquad (2)$$

Here $k = 2\pi/\lambda$ where λ is the wavelength of the light and s is the distance from the element in the aperture of area dS to the point at which the light is detected. The value of ks is the phase of the wave at the detector.

This expression applies in the special case for which the incident light is a plane wave so that all elements of the aperture develop a new wave in phase. The wavelets from each element arrive at the detector with different phases because the transit time from that element to the detector varies over the aperture. If we collect all the light going into a particular direction after the aperture by observing the sum of the wavelets at an infinite distance away then this integral may be evaluated exactly in cases where the boundary of the opening is a rectangle or a circle. This special case is called Fraunhofer diffraction. For a lens forming an image of a distant star, it describes the image of the star in the focal plane because a perfect lens collects all the light going into a direction and brings it to focus in a single point. Both circular and rectangular apertures have exact solutions that will enable the prediction of the angular resolution of a telescope.

4 Rectangular Aperture

We will do the rectangle first because the integration is more familiar. Since we are interested only in the change in the phase of the wave coming from different elements we can use

$$U(P) = C \int_{-a}^{+a} \int_{-b}^{+b} \exp(-ik(px + qy)) dx dy \tag{3}$$

where x and y are within the aperture. For z along the optical axis, we use θ and ϕ to represent the angles of the light propagation direction measured in the (x, z) and (y, z) planes, and define $p = \sin(\theta)$ and $q = \sin(\phi)$. For the small angles encountered in astronomy, p and q are nearly equal to the angles to the optical axis in radians.

The two integrals are independent and separable. Each one is of this form:

$$\int_{-a}^{+a} \exp(-ik(px))dx = -\frac{1}{ikp}(\exp(-ikpa) - \exp(ikpa)) \tag{4}$$

$$= 2\sin(kpa)/(kp) \tag{5}$$

The intensity of the light at the focus is proportional to the square of the amplitude and is given by

$$I(P) = |U(P)|^2 \tag{6}$$

$$I(P) = I_0 \,(\sin(kpa)/(kpa))^2 (\sin(kqb)/(kqb))^2 \tag{7}$$

where I_0 is the intensity at the center.

The function $\sin(\beta)/\beta$ is 1 when $\beta = 0$ and is 0 when $\beta = \pi$. It reduces in amplitude with increasing β, oscillating as it goes. This behavior produces the diffraction fringes with maxima nearly uniformly spaced but of diminishing significance at large β. The zeros are at multiples of π where the sin is zero:

$$kpa = \pm m\pi \tag{8}$$

$$kqb = \pm n\pi \tag{9}$$

with m and n integers 1,2,3,...

The first minimum is at $n = 1$ and the angle given by

$$p = \pi/ka \tag{10}$$

$$p = \lambda/2a \tag{11}$$

$$\theta \approx \lambda/2a \tag{12}$$

which is to say that the angle of the beam to the first minimum is the wavelength of the light divided by the aperture.

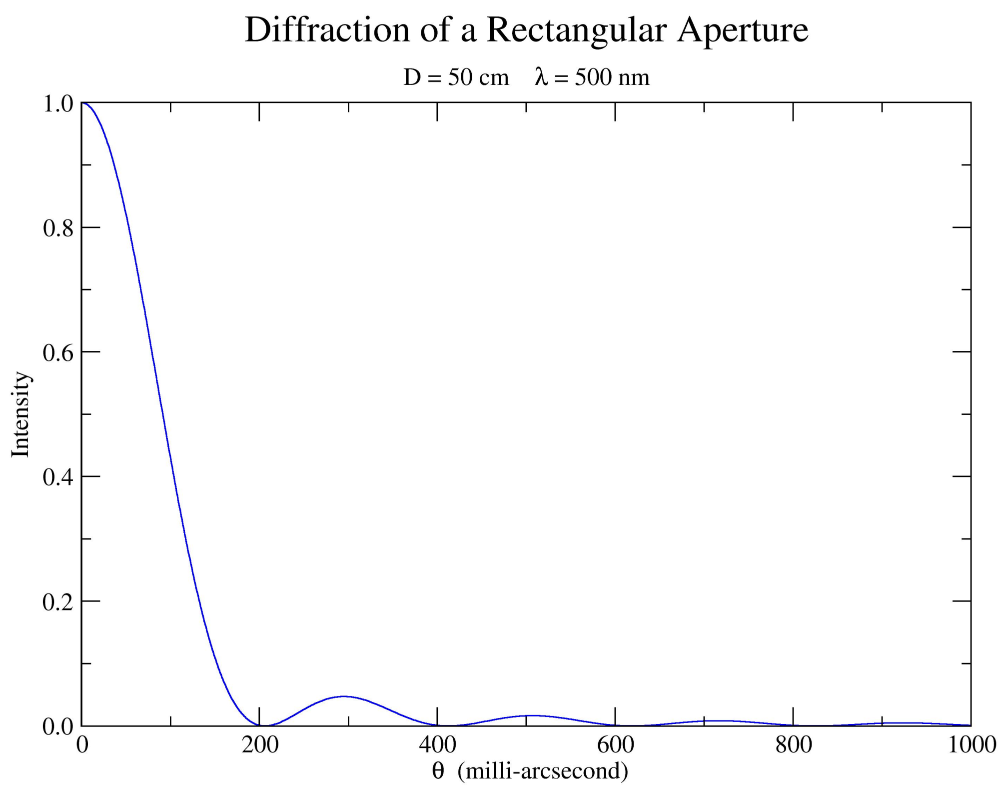

Figure 10: Diffraction of a 50 cm wide rectangular aperture at 500 nm.

5 Circular Aperture

In the case of a circular aperture the symmetry allows integration in only one variable. The equation for the amplitude is found by using

$$\begin{aligned} x &= \rho\cos(\eta) & (13)\\ y &= \rho\sin(\eta) & (14)\\ p &= w\cos(\psi) & (15)\\ q &= w\sin(\psi) & (16) \end{aligned}$$

where ρ and η are the radius and azimuthal angle within the circular aperture, and where $w = \sin(\theta)$ and ψ are measured in the image space. The angle ψ, like η, is measured azimuthally around the optical axis. The angle θ is measured from the optical axis to the direction of propagation and is analogous to θ in the rectangular case above.

With these substitutions, the amplitude in the focus in the direction P, defined by θ and ψ, is

$$U(P) = C\int_0^a\int_0^{2\pi}\exp(-ik\rho w\cos(\eta-\psi))\rho d\rho d\eta \tag{17}$$

The integral representation of the Bessel function is

$$J_n(x) = \frac{i^{-n}}{2\pi}\int_0^{2\pi}\exp(ix\cos(a))\exp(ina)da \tag{18}$$

leading to

$$\begin{aligned} U(P) &= 2\pi C\int_0^a J_0(k\rho w)\rho d\rho & (19)\\ &= C\pi a^2\left(\frac{2J_1(kaw)}{kaw}\right) & (20) \end{aligned}$$

The intensity for a circular aperture is

$$I(P) = I_0\left(\frac{2J_1(kaw)}{kaw}\right)^2 \tag{21}$$

As with the rectangular aperture, this is a central peak surrounded by rings of diminishing intensity at larger radii. The first zero of $J_1(x)/x$ is at $x = 1.220\pi$. Thus the first dark ring has a radius

$$\begin{aligned} w &= 1.220\pi/ka & (22)\\ &= 1.220\lambda/2a & (23) \end{aligned}$$

The angle from the axis to the first dark ring is $1.22\lambda/D$ where D is the diameter of the aperture. As a rule of thumb, for visible light, this angle is approximately 1 arcsecond for an aperture of 10 cm

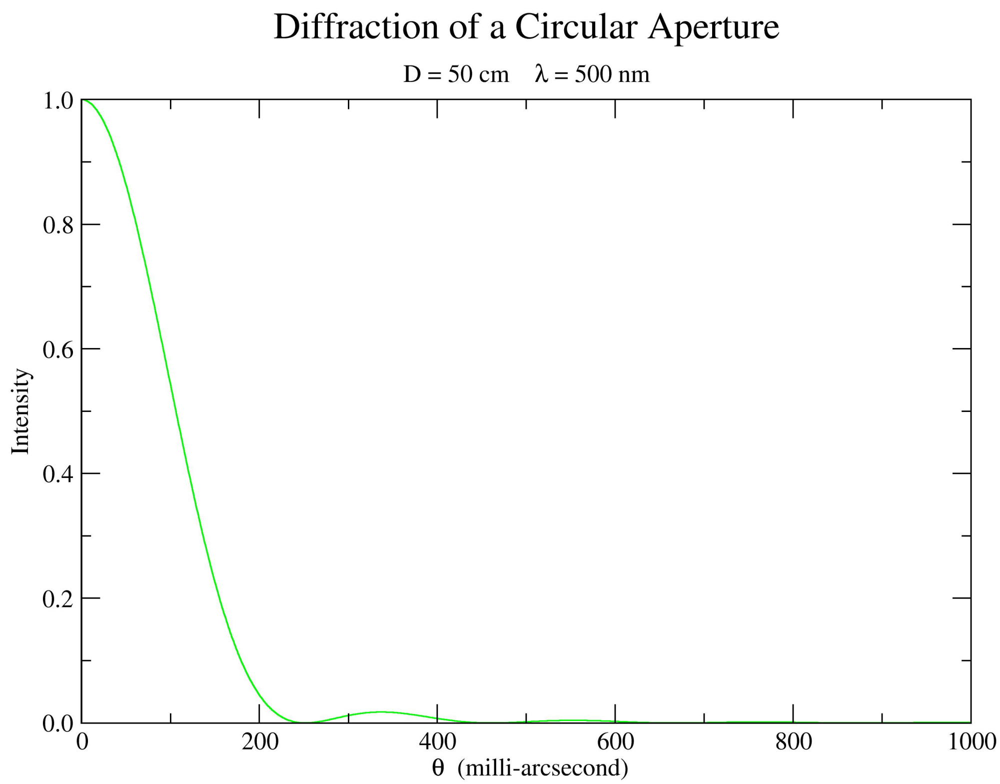

Figure 11: Diffraction of a 50 cm diameter circular aperture at 500 nm.

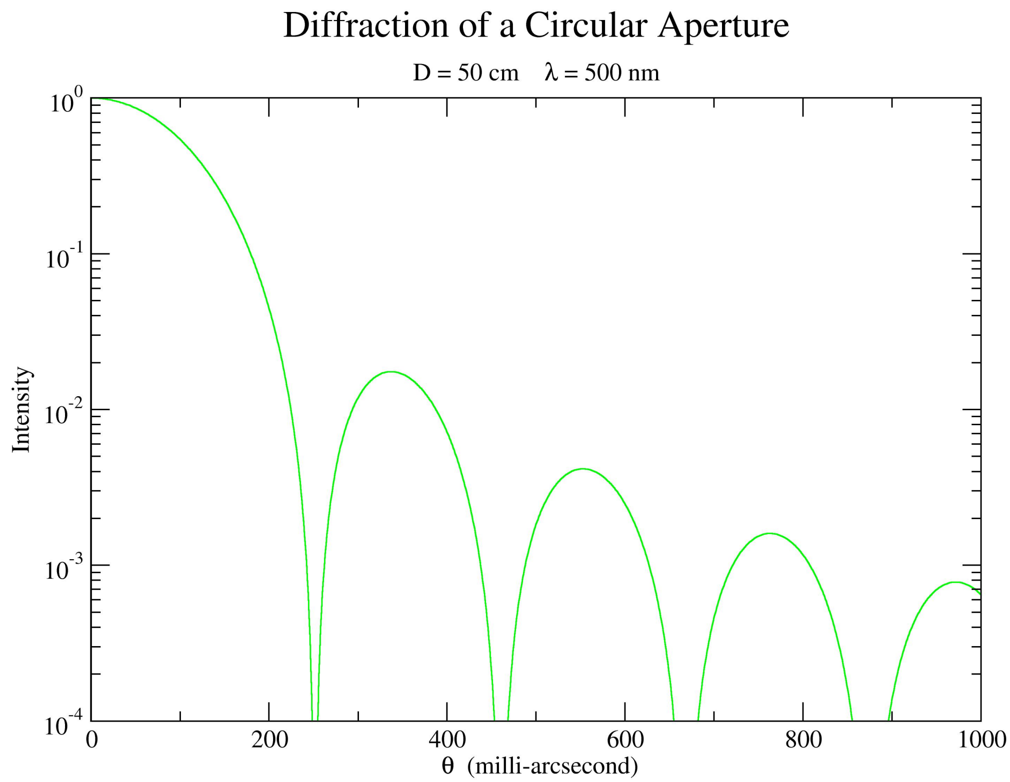

Figure 12: Logarithmic view of the point spread function for a diffraction-limited 50 cm diameter telescope at 500 nm.

6 Example

As an example consider a telescope with a diameter of 0.5 meters (50 cm, 20 inches). Its ideal performance is independent of its focal length and determined only by the diffraction pattern generated by its entrance aperture. There is a small difference in whether the aperture is circular or square because the minima are at slightly different angles, with the circular pattern about 20% larger. The first minimum is at about 220 milli-arcseconds. At 1 arcsecond the maximum diffracted light is about 10^{-3} of the peak.

A telescope such as this could, in principle resolve two stars approximately 220 mas apart. If a second star were centered on the zero of the first, it is simple to show that there would be a small discernable dip in the intensity of the sum of the two patterns that would make the second star detectable. However, if the second star were much fainter than the first, it would be much harder to detect unless it were farther away. This "point spread function" decreases by about 1000 times from the peak at 1 arcsecond.

7 Epsilon Lyrae

The star system ϵ Lyrae is seen by the unaided eye as a bright star in Lyra, and in binoculars or a small telescope as a double star with a separation of 173 arcseconds. Each of the stars is itself double:

ϵ_1 Combined magnitude 4.7, two stars of magnitude 5.0 and 6.1 separated by 2.39" at a position angle of 348 degrees, orbiting with a period of 1725 years.

ϵ_2 Combined magnitude 4.6, two stars of magnitude 5.2 and 5.5 separated by 2.37" at a position angle of 78 degrees, orbiting with a period of 724 years.

The star system is about 162 light years (49.7 parsecs). Note that a parsec is the distance at which 1 astronomical unit subtends an angle of 1 arcsecond.

The individual stars are clearly resolved in any small telescope with sufficient magnification. In a large telescope, however, atmospheric turbulence distorts the wavefront of the light from the stars, and creates a dynamic image that may blur these details when the "seeing" is poor.

References

[1] Richard Hedrick. *Planewave Instruments*. 2021. URL: https://planewave.com/ (visited on 09/19/2021).

[2] Max Born and Emil Wolf. *Principles of Optics*. 7th ed. Cambridge: Cambridge University Press, 1999. ISBN: 0-521-642221.

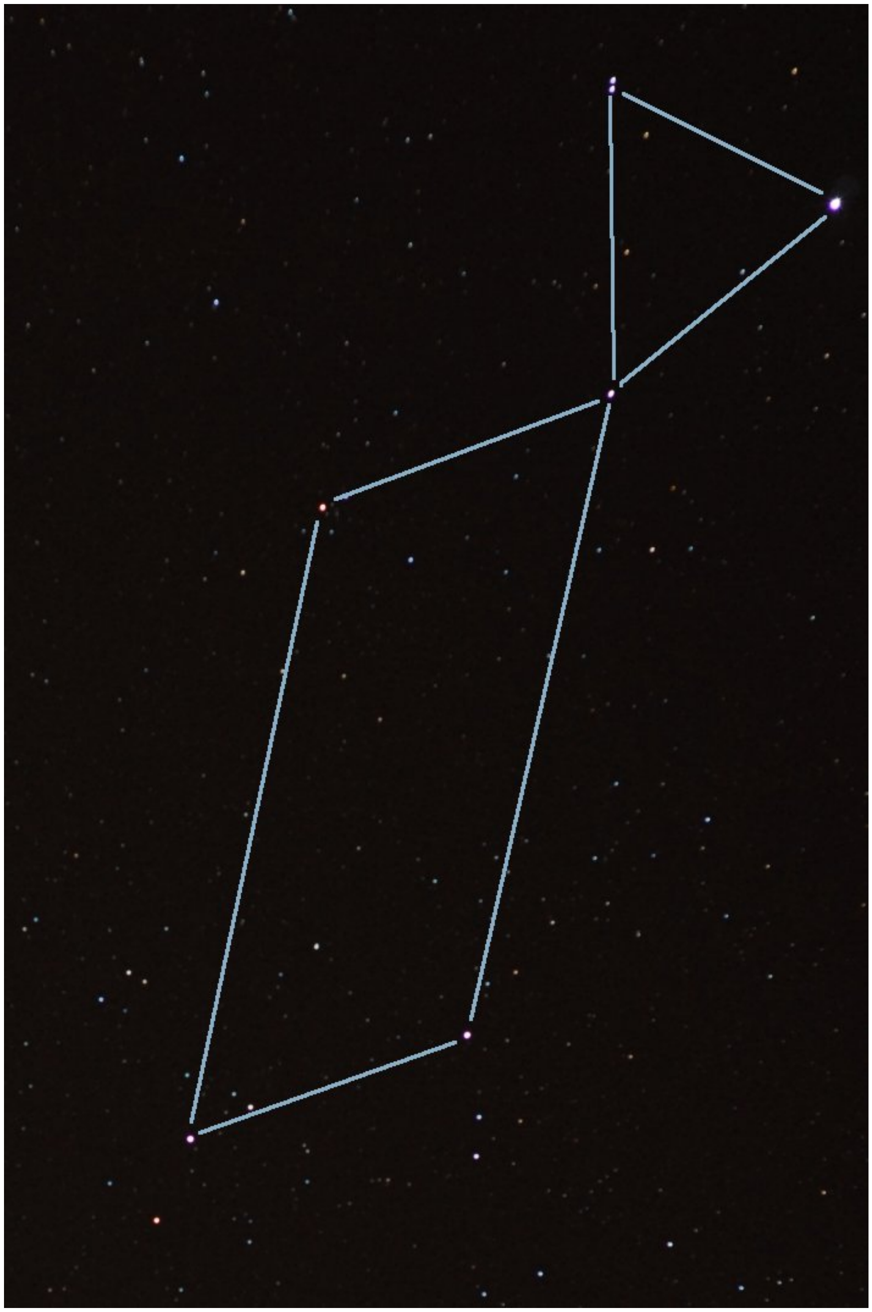

Figure 13: The constellation of Lyra, overhead in the northern hemisphere night sky at sunset in the fall. This image was recorded with a Nikon digital SLR, and a 3 second exposure with an 85 mm focal length lens at f/1.8. The bright star at the upper right is Vega, α Lyrae, and ϵ Lyrae is the the double star at the top right.

Figure 14: ϵ Lyrae in a 0.2 second exposure with the 0.5 m corrected Dall-Kirkham telescope at Moore Observatory. Compare the stars in the figure with the magnitudes, separations, and orientations described above. The line between the two double stars is approximately N-S. North is on the left.

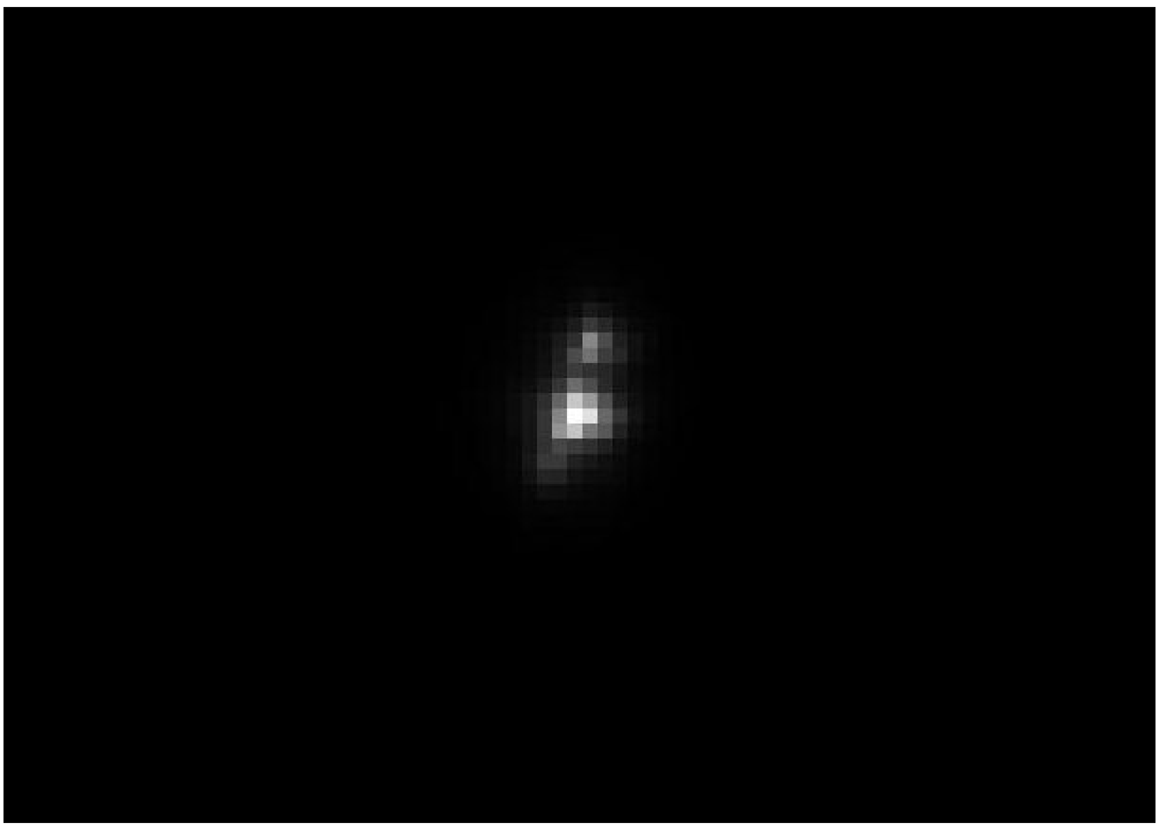

Figure 15: ϵ_1 Lyrae in a 0.03 second exposure with the 0.6 m Ritchie-Chretien telescope at Moore Observatory.

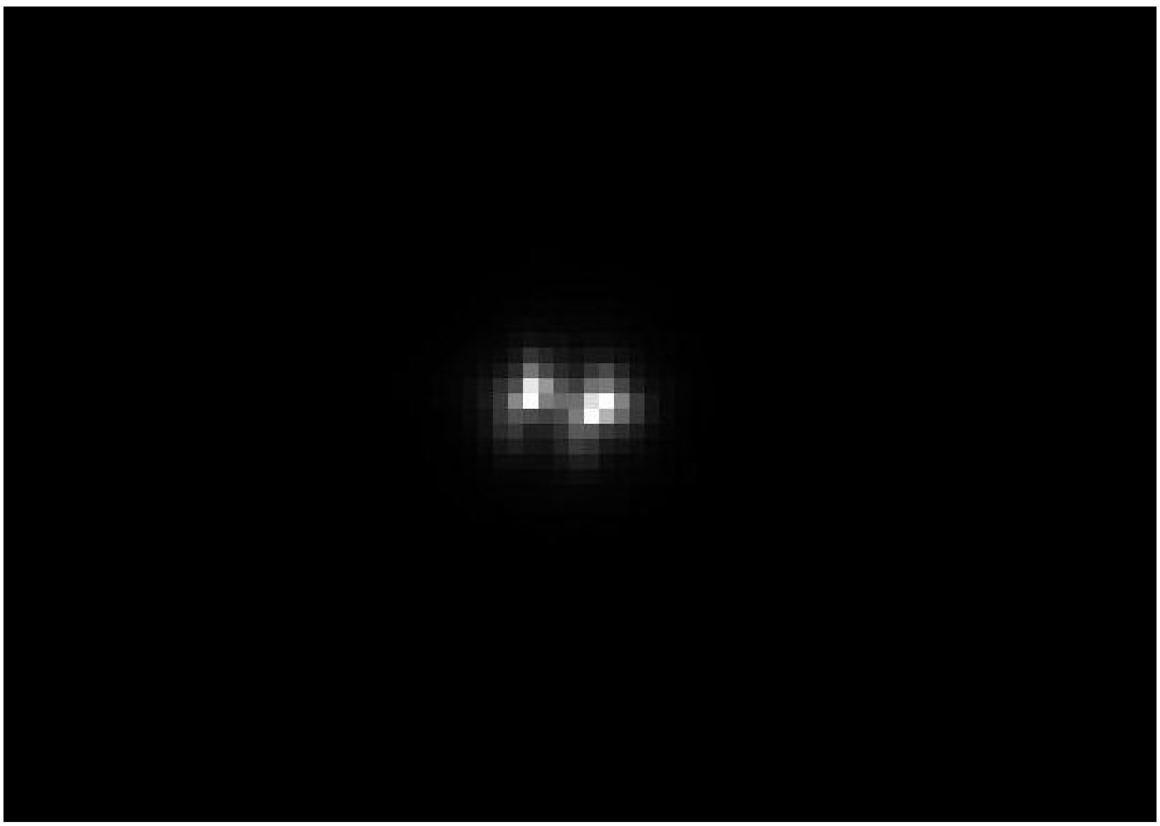

Figure 16: ϵ_2 Lyrae in a 0.03 second exposure with the 0.6 m Ritchie-Chretien telescope at Moore Observatory. You can see the resolution of the telescope in the individual sharply defined pixels, and the atmospheric blurring which takes light from the central diffraction maximum and spreads it around the image.